

Branching fraction of the isospin violating process $\phi \rightarrow \omega\pi^0$

C. Z. Yuan,* X. H. Mo,† and P. Wang‡

Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China

(Dated: September 2, 2010)

We examine the parametrization of the $e^+e^- \rightarrow \omega\pi^0$ cross section in the vicinity of the ϕ resonance and the extraction of the branching fraction of the isospin violating process $\phi \rightarrow \omega\pi^0$ from experimental data. We found that there are two possible solutions of the branching fraction, one is 4×10^{-5} , and the other is 7×10^{-3} . The latter is two orders of magnitude higher than the former, which is the commonly accepted one.

PACS numbers: 14.40.Be, 13.25.Gv, 13.66.Bc

I. INTRODUCTION

As has been pointed out in a recent study [1], there are many cases where multiple solutions are found in fitting one dimensional distribution with the coherent sum of several amplitudes and free relative phase between them. The fit to the $e^+e^- \rightarrow \omega\pi^0$ cross sections in the vicinity of the ϕ resonance was shown as an example of the existence of the two solutions and how large the difference could be between them.

However, Ref. [1] found these two solutions only through a fit to the experimental data, which causes suspicion that the two solutions may due to the statistical fluctuation or other reasons associated with the data handling or fitting procedure or something else. In this brief report, we show mathematically that two solutions exist in the parametrization of the cross section used in the original publication [2]; and the second solution can be obtained analytically from the solution reported in the literature without doing fit to the experimental data.

II. DERIVE OF THE SECOND SOLUTION

The cross section of $e^+e^- \rightarrow \omega\pi^0$ as a function of the center-of-mass energy, \sqrt{s} , is parameterized as

$$\sigma(\sqrt{s}) = \sigma_{nr}(\sqrt{s}) \cdot \left| 1 - Z \frac{M_\phi \Gamma_\phi}{D_\phi(\sqrt{s})} \right|^2 \quad (1)$$

in Ref. [2], where $\sigma_{nr}(\sqrt{s}) = \sigma_0 + \sigma'(\sqrt{s} - M_\phi)$ is the bare cross section for the non-resonant process, parameterized as a linear function of \sqrt{s} ; M_ϕ , Γ_ϕ , and $D_\phi = M_\phi^2 - s - iM_\phi\Gamma_\phi$ are the mass, the width, and the inverse propagator of the ϕ meson, respectively. Here Z is a complex number which depicts the interference effect. Conventionally, the real and imaginary parts of Z are denoted as $\Re(Z)$ and $\Im(Z)$, respectively.

If we write

$$G(s, Z) = 1 - Z \frac{M_\phi \Gamma_\phi}{D_\phi(\sqrt{s})}, \quad (2)$$

then in the complex-parameter space (denoted by a complex number Z'), we want to figure out all possible parameters which can satisfy the following relation

$$|G(s, Z)|^2 = |G(s, Z')|^2. \quad (3)$$

Notice that the above relation is to be true for any s , it should be true for some special values of s . If we firstly take a special value of s which satisfies $M_\phi^2 - s = 0$, then we obtain

$$|1 - iZ|^2 = |1 - iZ'|^2, \quad (4)$$

or

$$|Z|^2 + 2\Im(Z) = |Z'|^2 + 2\Im(Z'). \quad (5)$$

Secondly, we take another special value of s which satisfies $M_\phi^2 - s = M_\phi\Gamma_\phi$, we obtain

$$\left| 1 - \frac{1+i}{2}Z \right|^2 = \left| 1 - \frac{1+i}{2}Z' \right|^2, \quad (6)$$

or

$$|Z|^2 - 2\Re(Z) + 2\Im(Z) = |Z'|^2 - 2\Re(Z') + 2\Im(Z'). \quad (7)$$

Subtraction of Eq. (5) from Eq. (7) yields

$$\Re(Z') = \Re(Z). \quad (8)$$

With this equality, Eq. (5) is recast as

$$[1 + \Im(Z')]^2 = [1 + \Im(Z)]^2, \quad (9)$$

by virtue of which one gets either $\Im(Z') = \Im(Z)$ or $\Im(Z') = -2 - \Im(Z)$. As a summary, we have two sets of solutions:

$$\begin{aligned} \Re(Z') &= \Re(Z), \\ \Im(Z') &= \Im(Z); \end{aligned} \quad (10)$$

and

$$\begin{aligned} \Re(Z') &= \Re(Z), \\ \Im(Z') &= -2 - \Im(Z). \end{aligned} \quad (11)$$

*Electronic address: yuancz@ihep.ac.cn

†Electronic address: moxh@ihep.ac.cn

‡Electronic address: wangp@ihep.ac.cn

It is readily to check that the above two sets of solutions are true for the relation (3) for any s . Obviously, the first set of solution is trivial which can be expected intuitively. However, the second set of solution is fairly interesting which is firstly obtained analytically. More interesting thing is, according to the Eqs. (10) and (11), the second set of solution can be obtained from the first one. Both solutions describe the experimental data identically well and one can not distinguish them purely from the experimental data. Therefore we conclude that if the cross section of $e^+e^- \rightarrow \omega\pi^0$ as a function of the center-of-mass energy is parameterized as Eq. (1), there must be two sets of solutions of the interference parameter Z .

One remark on our mathematical analysis. More generally we write $G(s, Z)$ in the form

$$G(s, Z) = 1 + ZF(s), \quad (12)$$

with $F(s)$ being a complex function depending on s . If two special values of s are taken, from relation (3), we obtain two quadratic equations with two unknowns. Generally speaking, there should be four sets of solutions. The existence of the exact two sets of solutions of our example indicates that the number of sets of solutions depends strongly on the form of $F(s)$.

III. EXPERIMENTAL CONFIRMATION

From Ref. [2], $\Re(Z) = 0.106$ and $\Im(Z) = -0.103$ are acquired from a fit to the experimental data in the $\omega \rightarrow \pi^+\pi^-\pi^0$ decay mode. In the light of Eq. (11), the second set of solution can be acquired immediately, i.e. $\Re(Z) = 0.106$ and $\Im(Z) = -1.897$.

It is interesting to compare these results with those obtained from a fit to the experimental data [1]. From Table 2 of Ref. [1], we found that the real parts of the two solutions are identical, while the imaginary parts sum up to a number slightly different from -2 . A check of the fit results showed in Ref. [1] indicates that the -1.90 is

indeed from a rounding of -1.897 . Keeping one more digit, we find the sum of the imaginary parts is exactly -2 . Both the real parts and the imaginary parts agree perfectly between the fit results and the analytical evaluation.

IV. SUMMARY AND DISCUSSIONS

We showed above that there must be two solutions in extracting the branching fraction of $\phi \rightarrow \omega\pi^0$ with the parametrization of the $e^+e^- \rightarrow \omega\pi^0$ cross section around the ϕ resonance in Ref. [2]. While the first solution corresponds to $\mathcal{B}(\phi \rightarrow \omega\pi^0) = 4 \times 10^{-5}$ as reported in Ref. [2], the second solution would be $\mathcal{B}(\phi \rightarrow \omega\pi^0) = 7 \times 10^{-3}$, which is two orders of magnitude higher than the first one.

One may need to check whether the parametrization of the cross section is meaningful or if there are further constraints to the parametrization or the parameters, in order to pick out the physics solution from the two-fold ambiguities.

It is worth pointing out that $\phi \rightarrow \omega\pi^0$ is an isospin violating process and thus should be small. The branching fraction reported in Ref. [2] is already large compared to theoretical calculations [3]. However, if the physics is the second solution showed above, we would find theoretical calculations are too low.

Acknowledgments

This work is supported in part by the National Natural Science Foundation of China (10775412, 10825524, 10935008), the Instrument Developing Project of the Chinese Academy of Sciences (YZ200713), Major State Basic Research Development Program (2009CB825203, 2009CB825206), and Knowledge Innovation Project of the Chinese Academy of Sciences (KJCX2-YW-N29).

-
- [1] C. Z. Yuan, X. H. Mo and P. Wang, arXiv:0911.4791 [hep-ph].
 [2] F. Ambrosino *et al.* [KLOE collaboration], Phys. Lett. B **669**, 223 (2008) [arXiv:0807.4909 [hep-ex]].

- [3] G. Li, Y. J. Zhang and Q. Zhao, J. Phys. G **36**, 085008 (2009) [arXiv:0803.3412 [hep-ph]], and references therein.